

Similar Figures



Getting the Idea

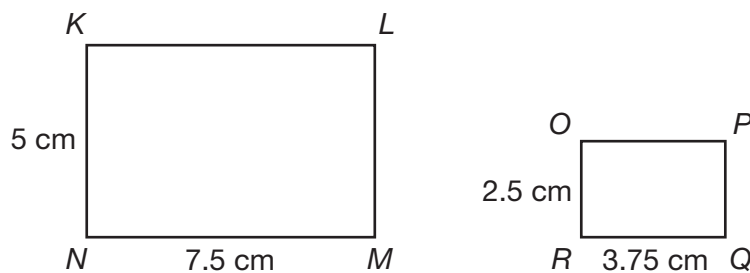
Similar figures have the same shape, but not necessarily the same size. For this reason, all **congruent figures** are also similar, but not all similar figures are congruent.

Similar figures have the following properties:

- Their **corresponding sides** have proportional lengths.
- Their **corresponding angles** are congruent.

Example 1

Are rectangles $KLMN$ and $OPQR$ similar?



Strategy Compare the angles and side lengths of the rectangles.

Step 1

Are the corresponding angles congruent?

All rectangles have four right angles, so the angles are congruent.

Step 2

Find the ratios of the lengths and widths of the rectangles.

$$\frac{NM}{RQ} = \frac{7.5}{3.75} \text{ and } \frac{KN}{OR} = \frac{5}{2.5}$$

Step 3

Determine whether the lengths are proportional.

$$\frac{7.5}{3.75} \stackrel{?}{=} \frac{5}{2.5}$$

Set up a proportion.

$$7.5 \times 2.5 \stackrel{?}{=} 3.75 \times 5$$

Cross multiply.

$$18.75 = 18.75 \checkmark$$

Find the cross products.

The cross-products are equal, so the lengths are proportional.

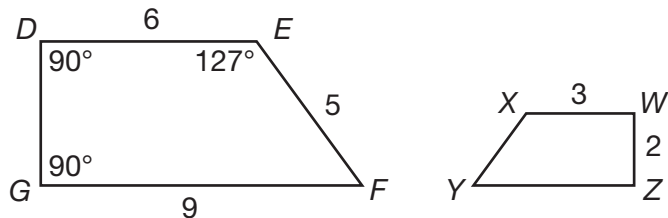
Solution

Rectangles $KLMN$ and $OPQR$ are similar because they have congruent angles and corresponding sides with proportional lengths.

Knowing that two polygons are similar allows you to find unknown lengths and angle measures.

Example 2

Trapezoid $DEFG$ is similar to trapezoid $WXYZ$ below.



What is the measure of $\angle X$ and what is the length of \overline{YZ} ?

Strategy Apply the properties of similar figures.

Step 1

Find the measure of $\angle X$.

$\angle X$ corresponds to $\angle E$.

Corresponding angles of similar figures are congruent.

Since the measure of $\angle E$ is 127° , the measure of $\angle X$ is also 127° .

Step 2

Write a ratio that includes the length of \overline{YZ} .

\overline{YZ} corresponds to \overline{FG} .

Let x represent the unknown length of side \overline{YZ} .

The ratio of the side lengths is: $\frac{FG}{YZ} = \frac{9}{x}$.

Step 3

Write another ratio that compares the lengths of corresponding sides.

The only corresponding sides that are both labeled are \overline{DE} and \overline{WX} .

$$\frac{DE}{WX} = \frac{6}{3}$$

Step 4

Set up the proportion and solve for x .

$$\frac{DE}{WX} = \frac{FG}{YZ}$$

$$\frac{6}{3} = \frac{9}{x}$$

$$6 \times x = 3 \times 9$$

$$6x = 27$$

$$\frac{6x}{6} = \frac{27}{6}$$

$$x = 4.5$$

So, $YZ = 4.5$ units.

Solution The measure of $\angle X$ is 127° , and the length of \overline{YZ} is 4.5 units.

You can use properties of similar triangles to find lengths using indirect measurement.

Example 3

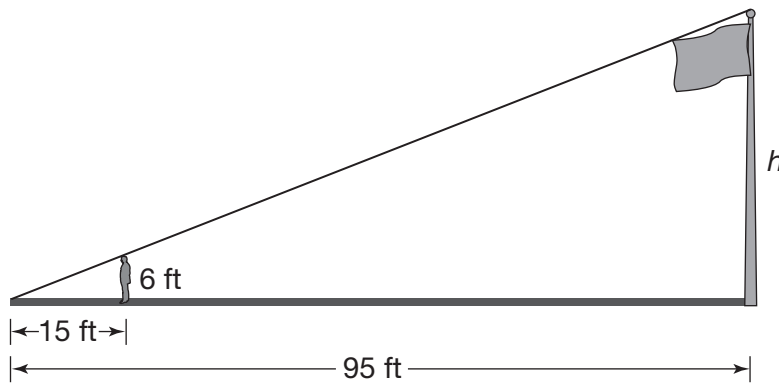
Nate is 6 feet tall and casts a shadow 15 feet long. At the same time, a nearby flagpole casts a shadow 95 feet long. What is the height of the flagpole?

Strategy Use similar triangles to set up a proportion.

Step 1

Draw a diagram to understand the problem.

The rays of the sun, the shadows, and the heights form a pair of similar right triangles.



Step 2

Set up a proportion using the similar triangles.

$$\frac{\text{flagpole's height}}{\text{length of flagpole's shadow}} = \frac{\text{Nate's height}}{\text{length of Nate's shadow}}$$
$$\frac{h}{95} = \frac{6}{15}$$

Step 3

Solve the proportion.

$$\frac{h}{95} = \frac{6}{15}$$
$$15 \times h = 95 \times 6$$
$$15h = 570$$
$$\frac{15h}{15} = \frac{570}{15}$$
$$h = 38$$

Solution The height of the flagpole is 38 feet.



Coached Example

Justin has two rectangular photo prints that are similar. The length of the smaller print is 5 inches and the width is 3 inches. The length of the larger print is 20 inches. What is the width of the larger print?

Use what you know about similar rectangles.

The lengths of two similar rectangles are corresponding sides.

The ratio of the lengths of those two prints is:

$$\frac{\text{length of smaller print}}{\text{length of larger print}} = \underline{\hspace{2cm}}$$

Let x represent the unknown width of the larger print.

The ratio of the widths of those two prints is:

$$\frac{\text{width of smaller print}}{\text{width of larger print}} = \underline{\hspace{2cm}}$$

Set up a proportion and solve for x .

$$\frac{\text{length of smaller print}}{\text{length of larger print}} = \frac{\text{width of smaller print}}{\text{width of larger print}}$$

The width of the larger print is _____ inches.

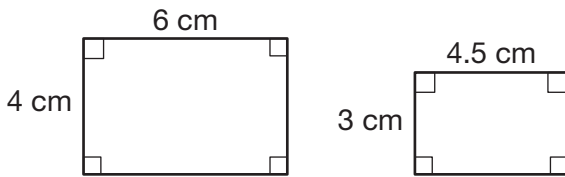


Lesson Practice

Choose the correct answer.

1. A gate is 3 feet tall and casts a shadow 5 feet long. At the same time, a nearby building casts a shadow 45 feet long. What is the height of the building?
A. 15 feet
B. 27 feet
C. 43 feet
D. 75 feet

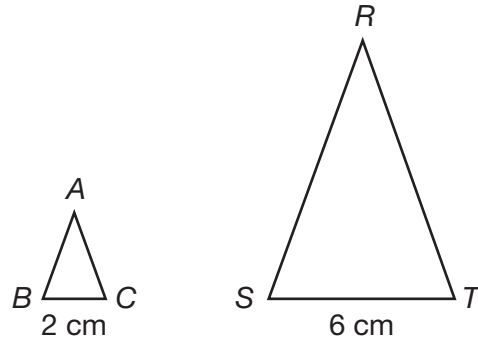
2. Which describes these two figures?



- A. congruent but not similar
 - B. neither similar nor congruent
 - C. similar but not congruent
 - D. similar and congruent
3. Paula casts a shadow 2 meters long at the same time a tree casts a shadow 28 meters long. The tree is 17.5 meters tall. How tall is Paula?
A. 1.25 meters
B. 1.75 meters
C. 2.5 meters
D. 2.8 meters

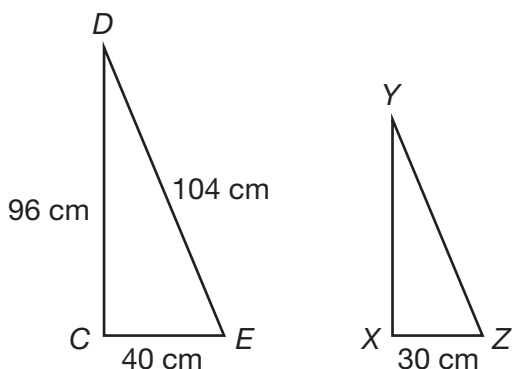
Use this diagram for questions 4 and 5.

In the diagram below, $\triangle ABC$ is similar to $\triangle RST$.



4. If the measure of $\angle A$ is 40° , then which angle must also measure 40° ?
A. $\angle C$
B. $\angle S$
C. $\angle T$
D. $\angle R$
5. If the length of RT is 12 centimeters, then what must be the length of \overline{AC} ?
A. 36 centimeters
B. 12 centimeters
C. 4 centimeters
D. 3 centimeters

6. If $\triangle CDE$ is similar to $\triangle XYZ$, what is the length of \overline{XY} ?

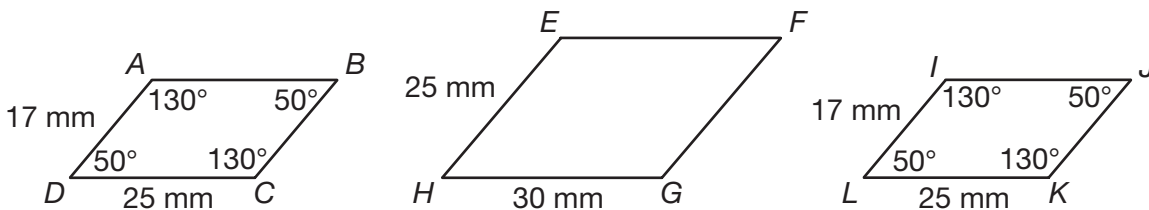


- A. 72 cm
- B. 78 cm
- C. 128 cm
- D. 139 cm

7. A house that is 16 feet tall casts a shadow 9 feet long. At the same time, a statue casts a shadow that is 8 feet long. Which is closest to the height of the statue?

- A. 4.5 feet
- B. 7 feet
- C. 14.2 feet
- D. 18 feet

8. Look at the parallelograms below.



- A. Are parallelograms $ABCD$ and $IJKL$ congruent? Are they similar? Explain your answer.

- B. Could parallelograms $ABCD$ and $EFGH$ be similar? Explain your answer.

9. Mariano is visiting Saguaro National Park. He is standing near three tall cacti. Mariano is 54 inches tall and casts a shadow that is 99 inches long. At the same time, the three cacti cast shadows of the given lengths. Use numbers from the box to make each statement true.

A cactus that casts a shadow of 495 inches is _____ inches tall.

A cactus that casts a shadow of 33 inches is _____ inches tall.

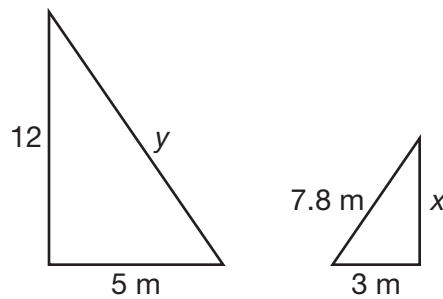
A cactus that casts a shadow of 154 inches is _____ inches tall.

10.8	84
18	162
34.7	270
60.5	282.3

10. Matryoshka dolls are Russian nesting dolls. To nest properly, the dolls must be similar figures. If the largest doll has a height of 15 centimeters and a base width of 6 centimeters, which set of dimensions could belong to a smaller nesting doll? Circle all that apply.

- A. 7.5 cm tall, 3 cm wide
- B. 7 cm tall, 2 cm wide
- C. 10 cm tall, 3 cm wide
- D. 12 cm tall, 5 cm wide
- E. 10 cm tall, 4 cm wide
- F. 3 cm tall, 1 cm wide
- G. 5 cm tall, 2 cm wide

11. The figures below are similar. Select True or False for each statement.



- A. The value of x is 7.2 m. True False
- B. The value of x is 1.9 m. True False
- C. The value of y is 13 m. True False
- D. The value of y is 4.7 m. True False

12. A puzzle company makes puzzles in several sizes that are all similar rectangles. Its 500-piece puzzle is 12 inches tall and 20 inches wide. Which set of dimensions could be another of the company's puzzles? Circle all that apply.

- A. 14 in. tall, 22 in. wide
- B. 15 in. tall, 25 in. wide
- C. 20 in. tall, 12 in. wide
- D. 9 in. tall, 15 in. wide
- E. 18 in. tall, 30 in. wide
- F. 6 in. tall, 10 in. wide
- G. 6 in. tall, 20 in. wide

13. A math textbook has a width of 9 inches and a height of 12 inches. If a history textbook has each of the given dimensions, are the two textbooks similar? Select Yes or No.

- A. width: 8 in., height: 11 in. Yes No
- B. width: 11 in., height: 15 in. Yes No
- C. width: 12 in., height: 16 in. Yes No
- D. width: $9\frac{3}{4}$ in., height: 13 in. Yes No

14. Lucinda is standing with her parents next to a railroad car for a picture. She is 42 inches tall and casts a shadow that is 14 inches long. At the same time, her parents and the railroad car cast shadows of the given lengths. Use numbers from the box to complete each statement.

If Lucinda's mom casts a shadow of 22 inches, she is _____ inches tall.

If Lucinda's dad casts a shadow of $24\frac{1}{6}$ inches, he is _____ inches tall.

If the train car casts a shadow of 52 inches, it is _____ inches tall.

$7\frac{1}{3}$	$72\frac{1}{2}$
$52\frac{1}{6}$	156
66	468